

# Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2018

November 1, 2018

## Part I

### A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**

As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**

The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

See Table 7 on page 9.

Table 1: Numbers in each class

	Number					Percentages %				
	2018	(2017)	(2016)	(2015)	(2014)	2018	(2017)	(2016)	(2015)	(2014)
I	53	(48)	(44)	(45)	(45)	56.99	(57.14)	(50.57)	(46.39)	(45.92)
II.1	26	(23)	(31)	(39)	(42)	27.96	(27.38)	(35.63)	(40.21)	(42.86)
II.2	13	(12)	(9)	(13)	(11)	13.98	(14.29)	(10.34)	(13.4)	(11.22)
III	1	(1)	(3)	(0)	(0)	1.08	(1.19)	(3.45)	(0)	(0)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(84)	(87)	(97)	(98)	100	(100)	(100)	(100)	(100)

## **B. Changes in examining methods and procedures currently under discussion or contemplated for the future**

None.

## **C. Notice of examination conventions for candidates**

The first notice to candidates was issued on 1st February 2018 and the second notice on 1st May 2018. These contain details of the examinations and assessments.

All notices and the examination conventions for 2018 examinations are on-line at <http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments>.

# **Part II**

## **A. General Comments on the Examination**

The examiners would like to thank in particular Gemma Proctor, Waldemar Schlackow and Charlotte Turner-Smith for their commitment and dedication in running the examination systems. We would also like to thank Nia Roderick, and the rest of the Academic Administration Team for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Chris Howls and Dr Jonathan Woolf for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting.

## **Timetable**

The examinations began on Monday 28th May and finished on Tuesday 12th June.

## **Medical certificates and other special circumstances**

The examiners were presented with factors affecting performance applications for five candidates.

## **Setting and checking of papers and marks processing**

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of

the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Gemma Proctor, Charlotte Turner-Smith and Hannah Harrison, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

### **Determination of University Standardised Marks**

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers  $N_1$ ,  $N_2$  and  $N_3$  are first computed for each paper:  $N_1$ ,  $N_2$  and  $N_3$  are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges  $[70, 100]$ ,  $[60, 69]$  and  $[0, 59]$ , respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map  $R \rightarrow U$  ( $R = \text{raw}$ ,  $U = \text{USM}$ ) which is piecewise linear. The

graph of the map consists of four line segments: by default these join the points  $(100, 100)$ ,  $P_1 = (C_1, 72)$ ,  $P_2 = (C_2, 57)$ ,  $P_3 = (C_3, 37)$ , and  $(0, 0)$ . The values of  $C_1$  and  $C_2$  are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by  $N_1$  and  $N_2$ , is the same as the I and II.1 proportion of USMs achieved on the paper. The value of  $C_3$  is set by the requirement that  $P_2P_3$  continued would intersect the  $U$  axis at  $U_0 = 10$ . Here the default choice of *corners* is given by  $U$ -values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points  $P_1, P_2, P_3$  by hand, so as to alter the map  $\text{raw} \rightarrow \text{USM}$ , to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper,  $P_1, P_2, P_3$  are the (possibly adjusted) positions of the corners above, which together with the end points  $(100, 100)$  and  $(0, 0)$  determine the piecewise linear map  $\text{raw} \rightarrow \text{USM}$ . The entries  $N_1, N_2, N_3$  give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of  $P_1, P_2, P_3$ .

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 2: Position of corners of piecewise linear function

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C1.1	(13, 37)	(39, 57)	(42.5, 72)		3	2	0
C1.2	(8.67, 37)	(25, 57)	(44, 72)		1	4	0
C1.3	(10, 37)	(23, 57)	(32.5, 72)		6	8	0
C1.4	(10, 37)	(22, 57)	(38.2, 72)		4	5	0
C2.1	(14, 37)	(23, 57)	(32, 72)		5	2	0
C2.2	(9, 37)	(23, 57)	(39, 72)		6	6	0
C2.3	(11, 37)	(19, 55)	(32, 72)		4	1	0
C2.4	(10, 37)	(19, 57)	(42, 72)		3	4	0
C2.5	(10, 37)	(27, 57)	(43, 72)		6	3	0
C2.6	(10, 37)	(40, 72)	(0, 0)		3	4	0
C2.7	(9.93, 37)	(27, 57)	(41, 70)		9	8	0
C3.1	(9, 37)	(20, 57)	(35.6, 72)		7	7	0
C3.2	(11, 37)	(21, 57)	(31, 72)		6	5	0
C3.3	(9, 37)	(21, 57)	(35.2, 72)		3	0	0
C3.4	(12.69, 27)	(27, 57)	(39, 72)		6	5	0
C3.5	(12.46, 37)	(25, 57)	(38.2, 72)		4	3	0
C3.6	(18.49, 37)	(30, 57)	(39, 72)		4	3	0
C3.7	(14, 37)	(22, 57)	(35, 72)		7	10	0
C3.8	(8.5, 37)	(17, 57)	(24, 72)		6	11	0
C4.1	(8.61, 37)	(23, 57)	(35, 72)		9	5	0
C4.2	(13.09, 37)	(21, 57)	(36, 72)		8	2	0
C4.3	(14, 37)	(28.5, 57)	(36, 72)		6	2	0
C4.6	(11.66, 37)	(26, 57)	(37, 72)		6	1	0
C4.8	(7.75, 37)	(20, 57)	(27, 72)		1	3	1
C5.1	(12, 37)	(29, 50)	(31.5, 57)		5	9	0
C5.2	(13, 37)	(22, 57)	(40, 72)		8	12	2
C5.3	(12.75, 37)	(22.2, 57)	(40, 72)		1	5	0
C5.4					10	15	1
C5.5	(16, 37)	(26, 57)	(35.6, 72)		13	18	3
C5.6	(9.65, 37)	(28, 57)	(36, 72)		11	16	2
C5.7	(7.81, 37)	(25, 57)	(35, 72)		4	11	2
C5.9	(11, 37)	(24, 57)	(33, 72)		4	7	0
C5.11	(9.93, 37)	(25, 57)	(41, 72)		7	11	2
C5.12	(11, 37)	(27, 57)	(40.8, 72)		11	12	1
C6.1	(12.63, 37)	(28, 57)	(37, 72)		11	11	2
C6.2	(10.74, 37)	(22, 57)	(35.2, 72)		12	15	2
C6.3	(11, 37)	(26, 57)	(38, 72)		7	14	2

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C6.4	(11, 37)	(25, 57)	(38, 72)		5	9	3
C7.4	(11, 37)	(26, 57)	(38, 72)		4	5	0
C7.5	(25, 50)	(32, 60)	(40, 70)		0	2	0
C7.6				0	2	0	
C8.1	(10.91, 37)	(21, 57)	(33, 72)		7	3	1
C8.2	(12.58, 37)	(27, 57)	(35, 72)		6	3	0
C8.3	(16, 37)	(28, 57)	(37, 72)		10	14	0
C8.4	(13, 37)	(29, 57)	(38, 72)		7	16	0
SC1	(16.88, 37)	(35, 57)	(42, 72)		8	21	0
SC2	(13.32, 37)	(23.2, 57)	(42, 70)		6	21	2
SC4	(10, 37)	(20, 57)	(30.5, 72)		9	16	2
SC5	(9.07, 37)	(21, 57)	(36, 72)		6	17	0
SC6	(9, 37)	(24, 57)	(34, 72)		4	13	0
SC7	(9.70, 37)	(24, 57)	(37, 72)		4	15	0

Table 6 on page 8 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 4: Percentile table for overall USMs

Av USM	Rank	Candidates with this USM or above	%
91	1	1	1.08
90	2	2	2.15
89	3	3	3.23
88	4	5	5.38
87	6	6	6.45
86	7	7	7.53
84	8	8	8.6
83	9	10	10.75
81	11	11	11.83
80	12	12	12.9
79	13	13	13.98
78	14	15	16.13
77	16	18	19.35
76	19	21	22.58
75	22	25	26.88
74	26	31	33.33
73	32	38	40.86
72	39	43	46.24
71	44	47	50.54
70	48	53	56.99
69	54	54	58.06
68	55	56	60.22
67	57	62	66.67
66	63	65	69.89
65	66	67	72.04
63	68	71	76.34

Av USM	Rank	Candidates with this USM or above	%
62	72	75	80.65
61	76	76	81.72
60	77	79	84.95
59	80	80	86.02
57	81	82	88.17
56	83	85	91.4
55	86	87	93.55
54	88	88	94.62
52	89	90	96.77
51	91	92	98.92
45	93	93	100

## B. Equality and Diversity issues and breakdown of the results by gender

Table 6: Breakdown of results by gender

Class	Number								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	6	47	53	11	37	48	10	34	44
II.1	7	19	26	5	18	23	10	21	31
II.2	3	10	13	2	10	12	4	5	9
III	1	0	1	0	1	1	0	3	3
F	0	0	0	0	0	0	0	0	0
Total	17	76	93	18	66	84	24	63	87

  

Class	Percentage								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	35.29	61.84	56.99	61.11	56.06	57.14	41.67	53.97	50.57
II.1	41.18	25	27.96	27.78	27.27	27.38	41.67	33.33	35.63
II.2	17.65	13.16	13.98	11.11	15.15	14.29	16.67	7.94	10.34
III	5.88	0	1.08	0	1.52	1.19	0	4.76	3.45
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100



### C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	5	-	-	-	-
C1.2	5	-	-	-	-
C1.3	13	28.46	8.11	66.08	12.26
C1.4	9	32.22	10.99	68.44	15.84
C2.1	7	32.14	7.67	71.71	12.84
C2.2	12	32.75	8.32	67.17	10.77
C2.3	5	-	-	-	-
C2.4	7	28	15.34	60.43	19.03
C2.5	9	37.56	13.82	74.67	21.97
C2.6	7	36.43	11.52	71.71	18.06
C2.7	17	38.88	7.36	72.24	11.51
C3.1	14	30	11.31	67.36	16.4
C3.2	11	25	6.71	62.45	10.63
C3.3	3	-	-	-	-
C3.4	11	37.64	7.61	73.27	12.65
C3.5	7	37.14	7.97	70.86	15.97
C3.6	7	39.14	3.76	73.86	7.97
C3.7	17	32.06	8.92	68.47	15.17
C3.8	17	19.24	5.92	60.47	12.4
C3.9	4	-	-	-	-
C4.1	14	33	10.12	71.5	15.41
C4.2	10	34.2	6.99	72.5	15.41
C4.3	8	32	9.5	65.62	16.02
C4.6	7	35.14	7.49	70.86	12.99
C4.8	5	22.2	6.46	61	10.89
C5.1	14	31.86	8.56	60.64	14.43
C5.2	22	33.27	10.42	68.36	14.17
C5.3	6	35.33	6.83	69.33	7.5
C5.4	22	65.05	9.95	65.05	9.95
C5.5	34	32.94	5.44	68.29	9.48
C5.6	29	33.45	7.42	68.48	12.3
C5.7	17	28.65	8.13	63.59	11.75
C5.9	11	29.64	8.49	66.36	13.83
C5.11	20	35.7	9.81	70.3	14.62
C5.12	24	35.17	9.17	68.29	13.5
C6.1	22	33.95	5.59	67.59	9.55
C6.2	26	30.69	6.7	67.62	8.94
C6.3	22	32.64	9.31	66.45	13.61
C6.4	17	30.59	9.52	64.53	13.78

Paper	Number of Candidates	Avg StDev		Avg StDev	
		RAW	RAW	USM	USM
C7.4	9	33.89	10.09	68.89	15.62
C7.5	2	-	-	-	-
C7.6	2	-	-	-	-
C8.1	11	31	5.95	70.09	8.12
C8.2	9	35.11	8.88	72.44	16.09
C8.3	24	34.17	6	67.83	10.87
C8.4	21	36.48	7.25	71.14	12.51
SC1	12	43.42	3	78.08	9.02
SC2	12	38.5	6.87	71	8.96
SC4	9	26.11	11.42	64.11	18.71
SC5	5	-	-	-	-
SC6	2	-	-	-	-
SC7	1	-	-	-	-
SC8	3	-	-	-	-
CCS1	5	-	-	-	-
CCS2	6	86.5	7.01	86.5	7.01
CCS3	1	-	-	-	-
CCS4	1	-	-	-	-
CCD	43	78.09	9.20	-	-
COD	2	82	7.07	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

**Paper C1.1: Model Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.33	16.33	9.01	3	0
Q2	19.2	18	7.12	4	1
Q3	21.33	21.33	3.51	3	0

**Paper C1.2: Gödel's Incompleteness Theorems**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.66	17.66	4.16	3	0
Q2	15.75	15.75	5.12	4	0
Q3	21	21	4.35	3	0

**Paper C1.3: Analytic Topology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.55	16.55	3.57	9	0
Q2	5.83	7	4.99	5	1
Q3	15.5	15.5	2.96	12	0

### Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	4.95	8	0
Q2	15.66	15.66	5.70	9	0
Q3	25	25	7.34	1	0

### Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.83	14.83	2.63	6	0
Q2	17.33	17.33	5.08	6	0
Q3	16	16	8.48	2	0

### Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.55	17.55	2.12	9	0
Q2	16.36	16.36	5.20	11	0
Q3	13.75	13.75	5.12	4	0

### Paper C2.3: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.5	14.5	1.29	4	0
Q2	19.5	19.5	3.53	2	0
Q3	12.25	12.25	6.5	4	0

### Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	15	9.20	6	1
Q2	7.5	7.5	6.36	2	0
Q3	15.16	15.16	7.38	6	0

### Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.33	20.33	5.43	9	0
Q2	23.5	23.5	2.81	6	0
Q3	10.33	7	5.85	2	1

**Paper C2.6: Introduction to Schemes**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21	21	6.29	7	0
Q2	15.42	15.42	6.52	7	0

**Paper C2.7: Category Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.05	18.25	4.05	16	1
Q2	20.64	20.64	4.41	17	0
Q3	14	18	3.46	1	2

**Paper C3.1: Algebraic Topology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.07	15.07	5.88	14	0
Q2	10.42	12.8	7.13	5	2
Q3	14.8	16.11	7.08	9	1

**Paper C3.2: Geometric Group Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.43	11	0
Q2	7.36	7.3	4.12	10	1
Q3	15	15	-	1	0

**Paper C3.3: Differentiable Manifolds**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.66	18.5	6.65	2	1
Q2	19	22	6	2	1
Q3	14.66	19	8.08	2	1

**Paper C3.4: Algebraic Geometry**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.72	19.72	5.06	11	0
Q2	18.16	18.16	4.70	6	0
Q3	16	17.6	5.54	5	1

**Paper C3.5: Lie Groups**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.71	19.71	3.09	7	0
Q2	15	15	6.08	3	0
Q3	19.25	19.25	3.30	4	0

**Paper C3.6: Modular Forms**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.8	18.8	2.58	5	0
Q2	17	17	2.16	4	0
Q3	22.4	22.4	1.94	5	0

**Paper C3.7: Elliptic Curves**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.2	13.2	3.45	10	0
Q2	16.31	16.31	5.57	16	0
Q3	19	19	2.92	8	0

**Paper C3.8: Analytic Number Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.2	8.30	1.78	13	2
Q2	9.8	13.28	6.67	7	3
Q3	9	9	3.03	14	20

**Paper C4.1: Functional Analysis**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.41	16.41	4.88	12	0
Q2	16	16	4.30	9	0
Q3	17.28	17.28	7.93	7	0

**Paper C4.2: Linear Operators**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.5	14.5	4.79	4	0
Q2	19.22	19.22	2.72	9	0
Q3	15.37	15.85	4.40	7	1

**Paper C4.3: Functional Analytical Methods for PDEs**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.12	16.12	3.35	8	0
Q2	15.87	15.87	6.77	8	0

**Paper C4.6: Fixed Point Methods for Nonlinear PDEs**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.83	17.83	3.54	6	0
Q2	11.4	16.33	7.82	3	2
Q3	18	18	3.80	5	0

**Paper C4.8: Complex Analysis: Conformal Maps and Geometry**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.75	13.75	3.20	4	0
Q2	6	6		1	0
Q3	10	10	3.74	5	0

**Paper C5.1: Solid Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.5	20.5	2.75	10	0
Q2	13.5	13.5	4.50	12	0
Q3	13.16	13.16	5.67	6	0

**Paper C5.2: Elasticity and Plasticity**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10	10	2	6	0
Q2	16.09	16.09	4.84	22	1
Q3	19.87	19.87	4.78	16	0

**Paper C5.3: Statistical Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.6	16.6	4.82	5	0
Q2	15.5	15.5	7.59	4	0
Q3	22.33	22.33	3.78	3	0

**Paper C5.5: Perturbation Methods**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.17	14.17	4.18	28	0
Q2	12.2	16.14	7.94	7	3
Q3	18.48	18.48	3.27	33	0

**Paper C5.6: Applied Complex Variables**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.23	17.85	5.36	10	1
Q2	16.15	16.15	3.45	19	0
Q3	16.10	16.10	4.62	19	0

**Paper C5.7: Topics in Fluid Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.76	13.76	4.40	17	0
Q2	9.8	9.8	6.68	5	0
Q3	17	17	4.88	12	0



**Paper C5.9: Mathematical Mechanical Biology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.63	16.63	3.61	11	0
Q2	13	13	4.39	7	0
Q3	17.33	17.33	6.50	3	0

**Paper C5.11: Mathematical Geoscience**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	18	6.25	16	1
Q2	12.6	12.6	5.87	10	0
Q3	21.42	21.42	3.20	14	0

**Paper C5.12: Mathematical Physiology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11	11	3.63	6	0
Q2	18.76	18.76	5.58	21	0
Q3	18.28	18.28	3.53	21	0

**Paper C6.1: Numerical Linear Algebra**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.86	15.86	2.92	15	0
Q2	16	16.93	5.35	15	1
Q3	16.43	18.21	6.03	14	2

**Paper C6.2: Continuous Optimization**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.08	14.08	3.50	24	0
Q2	16.61	16.61	3.81	26	0
Q3	12.33	14	3.05	2	1

**Paper C6.3: Approximation of Functions**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.45	15.22	7.22	9	2
Q2	15	15	4.98	15	0
Q3	16.95	17.8	7.08	20	1

**Paper C6.4: Finite Element Methods for Partial Differential Equations**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11	11.5	3.42	8	1
Q2	16.29	16.29	4.98	17	0
Q3	14.54	16.77	7.39	9	2

**Paper C7.4: Introduction to Quantum Information**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.4	17.4	5.94	5	0
Q2	18	18	6.38	6	0
Q3	14	15.71	6.76	7	1

**Paper C7.5: General Relativity I**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10	10	2.82	1	0
Q2	17	17		2	0
Q3	18	18		1	0

**Paper C7.6: Relativity II**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23	23	0	1	0
Q2	15	15		2	0
Q3	18	18		1	0

**Paper C8.1: Stochastic Differential Equations**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.90	17.90	3.47	11	0
Q2	12.71	14.33	4.82	6	1
Q3	10.61	11.6	4.57	5	1

**Paper C8.2: Stochastic Analysis and PDEs**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.2	15.2	3.83	5	0
Q2	18.14	18.14	4.59	7	0
Q3	18.83	18.83	5.41	6	0

**Paper C8.3: Combinatorics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.71	16.15	3.19	13	1
Q2	17.44	17.44	3.45	18	0
Q3	17.41	17.41	4.30	17	0

**Paper C8.4: Probabilistic Combinatorics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.83	13.83	2.63	6	0
Q2	18.4	18.4	4.28	20	0
Q3	19.68	19.68	3.26	16	0

**Paper SC1: Stochastic Models in Mathematical Genetics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.54	21.54	2.42	11	0
Q2	21.4	21.4	2.70	5	0
Q3	22.12	22.12	2.41	8	0

**Paper SC2: Probability and Statistics for Network Analysis**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	3.86	10	0
Q2	20.2	20.2	5.00	10	0
Q3	18.75	18.75	3.86	4	0

**Paper SC4: Statistical Data Mining and Machine Learning**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.66	11.66	4.97	9	0
Q2	11.25	21	11.38	2	2
Q3	12.57	12.57	6.90	7	0

**Paper SC5: Advanced Simulation Methods**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.33	12.33	2.88	3	0
Q2	14.2	14.2	4.08	5	0
Q3	16.5	16.5	2.12	2	0

## Paper SC6: Graphical Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12	12		1	0
Q2	19	19		1	0
Q3	15.5	15.5	0.70	2	0

## Paper SC7: Bayes Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16		1	0
Q2	20	20		1	0

## D. Recommendations for Next Year's Examiners and Teaching Committee

None

## E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

### C1.1: Model Theory

#### Question 1

Question 1 The solutions displayed a clear understanding of the issues, both in giving the proof and in the examples. Points were taken off mostly for incorrect formulas (such as  $(\exists x)(\exists y)(x \leq y)$  in place of  $(\forall x)(\exists y)(x \leq y)$ ) or for clauses where a precise solution was not attempted.

Question 2 Most solutions showed a good understanding of the notions and the methods of proof. In part (c), a number of nice proofs were offered. More than one however tried unsuccessfully to argue without using the theory of types that the existence of elementary embeddings in both directions implies an isomorphism, either on general grounds or by clever arguments in the specific case.

Question 3 On the whole, the scripts showed an understanding of this connection between linear algebra and model theory. In part (a), a surprisingly recurring mistake was to prove the correct statement  $|\mathbb{I}| \cdot |\mathbb{Q}| + |\mathbb{V}|$  comparing the cardinalities of a basis  $\mathbb{I}$  with a vector space  $\mathbb{V}$ , using the incorrect statement that any element of  $\mathbb{V}$  is a scalar multiple of a single element of  $\mathbb{I}$ . In part (b), the majority described the types correctly but did not always prove correctly that there is a unique 1-type of nonzero elements. This can be done either using quantifier-elimination (which needs to be proved) or an automorphism argument. In (d) many did not notice the need to show that any embedding of a 1-dimensional vector space into another is elementary; again this can be done using quantifier-elimination.

## C1.2: Gödel's Incompleteness Theorems

Subject to the constraint that 14 is not divisible by 3, the questions chosen by the candidates were evenly distributed among the three questions. A number of candidates, including some who were not the very weakest, were not entirely secure in the definitions of key notions from the course, though still managed to get somewhere with questions that used those notions.

Question 1 covered two different topics. Parts (a) and (b) are on arithmetization of syntax, essential to the first part of the course, and parts (c) and (d) are on the notion of disjoint sets separated by a formula in a system, essential in the middle part of the course, for Rosser's Theorem, and the provability of diagonal equivalences (needed for the proof of the second incompleteness theorem), which did not appear directly on the exam. Two parts, (a) and (c), were straight bookwork. Part (a) was done well, or well enough, by everyone who did Q1; part (c), which was pure bookwork, was not uniformly well done, though everyone who did this question got some way with stating and proving this key theorem. The notion of weak separation did not occur in the course, but most who attempted the question were able to work with it.

Question 2 began with the first and second incompleteness theorems. These parts were mostly well done, though one candidate who did very well on the other parts of this question surprisingly did nothing more than restate part (b). The result in part (a) was covered in lectures by extending the proof of the first half of the first incompleteness theorem, but one candidate managed to establish the  $\omega$ -incompleteness result more directly, without going all the way through the proof of the first half of the first-incompleteness theorem. Parts (c), (d), (e) developed results that were new to the course, following from material that was covered in the course. Everyone who did this question got somewhere with these parts, and several candidates did very well.

Question 3 covered provability logic, beginning with the basis of it, Löb's theorem. The focus thereafter was on fixed points. Part (b) required establishing a fixed point by derivation in  $GL$ , which constitutes an abstract form of the derivation of the second incompleteness theorem from Löb's theorem, a topic in the course, but not covered in this way. A number of candidates did this very well, though not all. One candidate used a rule of  $\supset$ -Introduction for  $GL$  in a form which is false, though there is a valid form which can be derived. Part (c), proving that fixed points are provably equivalent, was easy, given what was to be taken as given, but required appeal to the closure of provability in  $GL$  under substitution, which several candidates did not think to cite. Part (d) was a combination of knowing which results from the course to cite and carrying through derivations, and was done well by some but not all who answered this questions, but the weaker answers still go some way with this result.

## C1.3: Analytic Topology

**Question 1** Most parts of question 1 were done well, with the most difficult part being the combination of the two techniques from the proofs of 'paracompact regular implies normal' and 'Lindelöf regular implies normal'. A lot of candidates only used one of them and hence did not manage to complete the proof.

**Question 2** Not a popular question. A lot of the attempted solutions misunderstood what exactly they were asked to show in part (a) and what they could assume. In part (b) a number of candidates tried to simply grow the  $X$ -open neighbourhoods of a point  $x$  to  $\beta X$ -open neighbourhoods and took points of these. However, this does not guarantee that the created sequence does in fact converge to  $x$ .

**Question 3** Generally well done, although a lot of candidates were inefficient in carrying out the set-arithmetic required. In part (b)(i) it is important to check that  $X$  is covered by the basis. In (b)(ii) and (iii) and in part (c) some candidates had trouble to distinguish between subsets of  $X$  and subsets of  $Cl(X)$  (i.e. sets of subsets of  $X$ ).

### C1.4: Axiomatic Set Theory

Generally, the correctness of the submitted solutions was high with very few mistakes made. Most marks were lost in not attempting parts of the questions.

**Question 1** Part (c) was trickier than expected, with only few candidates considering a suitable  $V_\gamma$  as a counterexample. Some candidates tried to argue that  $V$  itself was a counterexample, although it does of course satisfy **Replacement**.

Part (d) had some good solutions, although the  $\omega_1$  case lacked details. Even some those candidate who observe in the  $\omega$  case  $\omega$  exists by **Infinity** did not spot that the existence of  $\omega_1$  is a little more subtle.

**Question 2** In the proof that  $L$  satisfies **Powerset**, a number of candidates tried to consider  $L_{\alpha+1} \cap \mathcal{P}(x)$  (for  $x \in L_\alpha$ ) which does not work.

Although the given well-order  $\leq_L$  was often used in (b)(iii), most candidates did not think about it in (iv) to pick a witness for the existential formula which was definable without parameters. In (v) only very few candidates used the countability of  $D$  to deduce that correctly that  $\alpha \in \omega_1$  - most candidates only proved that  $\alpha \leq \omega_1$ .

**Question 3** Only very few candidates attempted question 3, but those who did got very high marks.

### C2.1: Lie Algebras

Question 1 was popular but part (b) had few correct answers, not many candidates thought to compare the dimension of  $[c(\mathfrak{I}), \mathfrak{g}]$  with  $c(\mathfrak{I})$ .

Question 2 was also popular and had many good answers, based on Cartan's criterion and Weyl's theorem.

Question 3 had few attempts. In (a) some candidates kept using the notion of generalized root spaces even though in this question the Cartan algebra is assumed abelian and semisimple, which greatly simplifies the computation.

## C2.2: Homological Algebra

The exam was overall a little bit too difficult (compared to e.g. the previous years). Nevertheless, I felt that the 3 questions were of roughly equal difficulty. The last part of each question was only attempted by a minority of the students. This refers, specifically, to part (e) of question 1, parts (c,iii) and (d,ii) of question 2, and part (d) of question 3.

## C2.3: Representation Theory of Semisimple Lie Algebras

**Question 1.** Part (a) was standard bookwork. In (b), a common mistake was to define the map  $\tilde{\tau}$  only on the basis of  $U(\mathfrak{g})$ . This does not prove that  $\tilde{\tau}$  is an algebra homomorphism, one needs to use the universal property in (i) for example. For (b)(ii), a mistake was to not use the Harish-Chandra projection: the question asked to show that  $Z(\mathfrak{g})$  is fixed *pointwise* not just preserved as a subalgebra. For (c), the expected solution was to look at the action of  $(1 - e)^\ell$  in order to “separate” the  $h^n$ 's.

**Question 2.** Parts (a) and (b) were standard. Some candidates forgot to check that the dimensions of weight spaces in category  $\mathcal{O}$  are finite (in order to deduce that the character is well defined). In part (c) it was expected to use the map defined by convolution by the Weyl denominator for example. In part (d), one had to use the classification of simple finite-dimensional modules via highest weights and argue that the numerator of the Weyl dimension formula increases if we “add” fundamental weights to the highest weight.

**Question 3.** Part (a) is standard. Part (b) is close to bookwork, but a common mistake was to say that this follows immediately from restrictions to the various  $\mathfrak{sl}_\alpha$ . There are some details to consider beyond this initial observation (see lecture notes). For part (c)(ii), it is easier if one determines the highest weight vectors, rather than to check the Weyl dimension formula.

## C2.4: Infinite Groups

**Q1.** This question was attempted by all the candidates. The majority provided an almost complete answer to the first question about the Ping-pong Lemma. A few were unable to suggest a generalization to  $n$  elements.

Question (b) was likewise attempted by all, some used the Ping-pong Lemma, others classical topological methods, all acceptable. For question (c) there were essentially two types of arguments, both explained in the Lecture Notes, and both were used.

**Q2.** This question was attempted by two candidates, only one of which was able to provide a reasonably complete answer. This was somehow surprising for a question that was rather concrete and focussed on very easy examples of wreath products. It is very likely that this is due to the fact that most candidates have not seen wreath products before this course.

**Q3.** This question was attempted by a large majority of candidates. Most could provide answers to the first three questions, related to material seen in lectures or Lecture Notes, though some mistakes or unclear arguments appeared here and there.

## C2.5: Non-Commutative Rings

Q1 was done well and most popular.

Q2 was done well by those who attempted it.

Q3 was least popular, despite arguably being the easiest.

## C2.6 Introduction to Schemes

Q1 (a). A fast way to produce an example of a non affine and non reduced scheme is to consider a variant of the scheme considered in Ex.1 of Sheet 3, replacing  $\mathbb{C}$  by a non reduced commutative ring. Most candidates saw this.

Q1 (b). If some stalk is non reduced then there is an open subset  $U$  of  $X$  and an  $f \in \Gamma(U, \mathcal{O}_U)$ , which is non zero and nilpotent. This follows from the definition of a stalk. The sheaf property of  $\mathcal{O}_X$  is not needed here, unlike what some candidates thought.

Q1 (c). If  $U$  is open in  $X$  and  $f \in \Gamma(U, \mathcal{O}_U)$  is nilpotent then the image of  $f$  in all the stalks of  $U$  vanishes. This implies that  $f$  vanishes because  $\mathcal{O}_U$  is a sheaf. Most candidates saw this. It is also possible to reduce this problem to the situation where  $X$  is affine and solve the problem using commutative algebra.

Q2 (a). One might consider a union of two hyperplanes in  $\mathbb{P}^2$  (the zero sections of  $X_0 \in \Gamma(\mathbb{P}^2, \mathcal{O}(1))$  and  $X_1 \in \Gamma(\mathbb{P}^2, \mathcal{O}(1))$ ) or simply the disjoint union of two copies of  $\mathbb{P}^1$ . Most candidates considered variants of the second example.

Q2 (b). If not, there is a non empty open affine subscheme  $U$  of  $Y$  such that  $f^{-1}(U) = \emptyset$  and thus  $\mathcal{O}_Y(U) \rightarrow (f_*(\mathcal{O}_X))(U) = \mathcal{O}_X(f^{-1}(U)) = 0$  is the zero map, which contradicts the assumption. One may note that the assumption that  $Y$  is affine is redundant. It was put there to ease a solution via commutative algebra, which is also possible. This is what several candidates did.

Q2 (c). If  $Y$  is not integral then there are elements  $a, b \neq 0$  such that  $a \cdot b = 0$ . The basic open subset  $D_a$  and  $D_b$  of  $Y$  are then disjoint and not empty and  $\Gamma(\mathcal{O}_X, f^{-1}(D_a \cup D_b))$  is not a domain, because  $f^{-1}(D_a \cup D_b) = f^{-1}(D_a) \cup f^{-1}(D_b)$ ,  $f^{-1}(D_a) \cap f^{-1}(D_b) = \emptyset$  and  $f^{-1}(D_a), f^{-1}(D_b) \neq \emptyset$  by the density assumption. This contradicts the assumption that  $X$  is integral. One may here again reduce to the situation where  $X$  is affine and treat the problem by commutative algebra. This is what some candidates did.

Q3 was not attempted by any candidate.

## C2.7 Category Theory

Very few candidates attempted Q3. The standard of answers to Q1 and Q2 was good. Very few candidates gave correct answers to Q1(c)(ii). Some candidates failed to realise in Q2(c) that the coequaliser is the quotient (set of equivalence classes).



### C3.1: Algebraic Topology

Question 1. All candidates attempted this question; the standard of answers was generally reasonable and included two perfect answers. (b) Most candidates here correctly applied the resolution procedure for Tor and Ext, but there were errors concerning more elementary matters of tensor products and homomorphisms of abelian groups. (c.i/ii) Most candidates correctly applied the universal coefficient theorems, but there were a number of elementary errors in computing kernels of maps of abelian groups. (c.iii) There were a few excellent answers here, though also many candidates failed to prove (or understand they needed to prove) that the manifold would have to be of dimension 2, before arguing from Poincare Duality, and also many candidates failed to apply Poincare Duality with  $\mathbb{Q}/\mathbb{Z}$  coefficients, despite the structure of the question indicating that was the appropriate path.

Question 2. Fewer candidates attempted this question, and half of those who did struggled with it. There was one strong answer. (b.iv) A couple candidates clearly identified a simple example, for instance the projective plane cross a circle, but some erroneously suggested the three-dimensional projective space. (c) Almost no one successfully identified that the space is homotopy equivalent to the wedge of six circles. Inexplicably a number of candidates failed to apply Lefschetz Duality, despite the structure of the question indicating that was the appropriate tool.

Question 3. Most candidates attempted this question; the standard was high overall and included one perfect answer. (b) Most candidates correctly identified these cocycles, reflecting either good geometric understanding or good algebraic computation; a very small number of candidates unfortunately computed using the genus one rather than genus two surface. (c) Most candidates correctly computed the products, though only some fully identified the Poincare dual classes as simplicial cycles, for instance via the cap product.

### C3.2 Geometric Group Theory

**Q1** This was a basic question about presentations and algorithmic problems. All students attempted this. Part a.i was done well. Some students had difficulties with part a.ii. However most students solved this considering homomorphisms to  $\mathbb{Z}_2$  or to a free group. Some used Tietze transformations giving lengthier proofs.

Surprising many candidates had difficulties with part bi. They failed to realize either that the presentation given was that of the quotient group  $G/F$  or they missed the fact the  $g$  is the identity in this quotient group iff  $g \in F$ .

Some students failed to do bii as they did not interpret correctly equality of words in  $G$ .

Quite a few candidates that did not do bi, bii went on to solve biii assuming the results of the previous part.

Some candidates had a valid idea of using homomorphisms for b.iv but mistakenly tried to list homomorphisms  $G \rightarrow S_n$  rather than  $S_n \rightarrow G$  which would have worked. Several students used a straightforward argument with words.

Only two candidates answered b.v.

**Q2** This was a question on amalgamated products and actions on trees attempted by most students.

Several candidates had difficulties with part a. They realized that it could be done using normal forms but they did not think of cyclically reduced normal forms. For the second part group actions on Trees were appropriately used, however several students claimed that  $H$  is free rather than a free product and many did not explain in detail why  $H$  is a free product.

In part b some candidates assumed that the tree  $T$  is finite - which was not assumed.

Quite a few candidates had the right geometric idea of constructing a translation axis and they got either partial or full credit for this when they gave a complete argument. Very few saw that commuting elements fix the same axis which was needed for the action of  $\mathbb{Z}^3$  on  $T$ .

Some students realized that they could use the results of part b for c and got either partial or full credit for this part.

**Q3** This question was attempted by only 2 students. It was on the last part of the course dealing with quasi-isometries and hyperbolic groups.

Both candidates did well answering most parts that were either bookwork or close to bookwork.

They had the right idea on how to show that  $G \times G$  has one end and got partial credit for this.

One candidate realized which triangles are not thin for the last part and got credit for this even though they failed to produce a complete solution.

### C3.3: Differentiable Manifolds

Question 1: A wide spread of marks, from very good, to candidates who apparently didnt understand the basics. For (a), almost everyone who answered correctly reproduced a proof from the notes that was unnecessarily complex, as it also gave  $f = 1$  near  $x$ .

Question 2: Again, a wide spread of marks. Some candidates tried incorrectly to use Cartans formula in (b). Parts (d) and (e) were found difficult, and no one used the hint in (e).

Question 3: Candidates did better on this question, as many of them were able to get close to full marks on (a)-(d). No one answered (e) correctly. The answer I was hoping for was this: fix a base point  $x_0$ . For any other point  $x$  in  $X$ , join  $x_0$  to  $x$  by a smooth path  $\gamma$ . The equation in (d) implies a second-order o.d.e. along  $\gamma$  on the coefficients  $i$  restricted to  $\gamma$ . Hence results on o.d.e.s imply  $i$  at  $x$  is determined by  $i$  and its first derivatives at  $x_0$ , and this holds for all  $x$  in  $X$ .

### C3.4: Algebraic Geometry

All students attempted Q1, and then the students split 50/50 on choosing Q2 or Q3.

Q1: In (d) there was some confusion among candidates trying to consider the two hypersurfaces given by the two defining equations, instead of noticing that  $Y$  was a subset of  $X$ , and noticing that  $X$  also contains the line given by the  $Z$  - axis.

Q2: Some slips in (a) caused by the fact that  $\dim X$  is  $\dim S(X) - 1$  (the drop in 1 is caused

by the presence of the irrelevant ideal). None of the candidates solved the second part of (b)(iv), one needed to consider the case when  $Q$  was a union of two lines.

Q3: In (a) candidates usually forgot to say that one picks an affine open neighbourhood of a point in the definition of regular function. In (b) most candidates just assumed that locally  $f, g$  belong to  $K[x, y]$ , instead of considering basic open sets where  $f, g$  live in a localisation of  $K[x, y]$  and then passing to  $K[x, y]$  by rewriting the fraction

### C3.5: Lie Groups

#### Question 1

This question was about the exponential map and the relation between Lie algebra and Lie group homomorphisms.

The earlier parts of the question were generally well done, and candidates showed a good understanding of the generation lemma. Some answers were too sketchy in showing that  $\exp$  was a local diffeomorphism around the identity. The last part (finding a Lie algebra homomorphism that did not integrate to a Lie group map) proved harder than expected, though a few candidates succeeded, either considering the group  $SO(3)$  or  $S^1$ .

#### Question 2

This question was on representations of  $SU(2)$  and characters,

Candidates understood the idea of restriction to a maximal torus but failed to take account of the Weyl-invariance. Surprisingly, nobody really got the decomposition of the tensor product in the final part, although it is an easy character calculation.

#### Question 3

This question was on Haar measure.

Candidates had a good grasp of the bookwork. The calculation in the final part, showing that a certain noncompact solvable group had essentially distinct left and right Haar measures and hence admitted no bi-invariant measure, proved more difficult but there were some very good answers here.

### C3.6: Modular Forms

The three questions were equally popular, with around two-thirds of candidates attempting each one. Probably the final parts of Question 1 were found the most challenging on the paper, but some students had near complete answers for this.

Question 1: Part (a) was done very well, though some students got the stabilisers wrong and others got confused on the calculations in (iii). Part (b) was mostly done correctly. In Part (c), (i) was fine, but most students struggled on (ii), (iii) and (iv), although (ii) was related to a problem on the homework sheets.

Question 2: Part (a) was done very well. Part (b) (i) was very similar to something in lectures, but most people had difficulty with this; (ii) was on the whole fine, and there were several correct solutions to (iii).

Question 3: Part (a) was very well done. Part (b) (i) was fine, but there were some confused answers to (ii) even though it was on homework sheets. Candidates had no problems with Part (c), which was quite pleasing as it was original so this showed they had a good understanding of the material.

### C3.7: Elliptic Curves

There were 19 candidates; 12 answered Q1, 18 answered Q2 and 8 answered Q3. In Q1, parts (a),(b) were well answered, but many candidates found part (c) difficult. In Q2(b), most candidates answered well the case when  $p$  is congruent to 2 mod 3, but very few completely answered the case when  $p$  is congruent to 1 mod 3. For Q2(c), about a third of the candidates wasted a lot of time trying to remove the  $X^2$  term from the curve, which is messy (it is easier just to leave the curve in its given form). Q3 was only answered by 8 candidates; most of those who answered it did so to a high standard.

### C3.8 Analytic Number Theory

Here is a question-by-question breakdown, with minor comments on modifications to the mark scheme, which were all in the form of further subdivisions of the existing allocations.

Q1. Almost no candidate was able to provide the estimate  $O(\log N)$  for  $\sum_{n=1}^N n^{-s}$ , where  $s = \sigma + it$  with  $\sigma \geq 1$ ; candidates do not seem to have a basic grasp of how exponentials behave with complex arguments, or even what the  $O()$  notation means; many candidates simply wrote that this was  $O(1)$ .

Q2. The attempts on this question were marginally better than on Q1, with a sprinkling of scores around 14 and one score of 22. However, the performance of candidates was a still a disappointment given that something very similar was done in lectures, and something almost identical was done on an example sheet. Candidates do not seem to have the basic ability to estimate the magnitude of quantities and to perform rough estimations by, for example, chopping up the domain into dyadic ranges - even though this was done several times in lectures in very similar situation.

Q3. In this question, some of the subparts were a little more independent of one another than in the other questions. A number of candidates (the majority) did (a) by appealing the the technique of Q1 (a), (b) seemed quite difficult, given that several similar examples were an early question on example sheet 2. Several candidates said they were going to show that  $\tau * \tau = 1_S * \tau^2$  directly, and then gave short but false arguments for this (for example by using the same dummy variable twice when expanding out a sum). This *can* be done directly, but it is not easy - probably harder than the official solution via Euler products. No candidates wrote down the correct formula for the residue of a function with a pole of order 4 at a point (part (e)). Only two candidates seemed to realise that they couldn't just compute  $\lim_{z \rightarrow a} (z - a)f(z)$ . However, this was not supposed to be an exam in complex

analysis and so in retrospect I should have provided the formula (which would then have made the question a computation).

#### **C4.1: Functional Analysis**

The exam was taken by 14 MMath candidates and one MTP candidate. It is pleasing to see that advanced courses in pure analysis continue to attract significant interest, especially among high-calibre students.

**Question 1.** Part (a) was straightforward bookwork. In part (b)(iv) only the stronger candidates realised that they could apply the Hahn-Banach theorem to the zero subspace. Part (c) led to a variety of outcomes, with some candidates declining my invitation to consider the diagonal subspace of  $X^n$  and instead offering an inductive argument based on part (b).

**Question 2.** While part (a) was generally well done, the arguments given in part (b) were often rather sloppy. Most candidates coped reasonably well with (c), and even those who struggled in parts (i) and (ii) tended to spot that (iii) was something of a gift.

**Question 3.** Candidates generally had little difficulty with the bookwork in part (a). In part (b)(ii) some candidates failed to appreciate that proving boundedness of  $T$  required an application of the uniform boundedness principle (or something similar). Part (c) received only a small number of convincing answers. In particular, nobody saw how to take advantage of the fact that the norms of the functionals in question are Riemann sums.

#### **C4.2: Linear Operators**

Q.1: This question was from the early part of the course, but it was less popular than I expected. The candidates found it quite hard.

Q.2: This question was on perturbation theory which had not been included in the course in previous years. The question was more popular than I expected, and it was answered very well except for the very last sub-part of (c) which required rewriting  $A+B$ .

Q.3: Parts (a) and (b) were generally answered well. Part (c) needed a variation of arguments that candidates had seen in earlier courses, and nobody gave a complete answer for it.

#### **C4.3: Functional Analytic Methods for PDEs**

In Question 1, parts (a),  $(a_i)$  and  $(a_{iii})$  are a bookwork and almost all student have done it well. In part  $(a_{ii})$ , many students proved that the required functional is a distribution but not all of them showed carefully that it is a singular distribution. Regarding part (b), the typical incomplete solution was to show that the statement is true for regular distributions. However, the question does not assume that the distribution is regular.

In part (c), there are two points: calculation of partial sums and passing to the limit in the sense of distributions for highly oscillating functions. In the most of papers, students took care only on one of the above points.

Question 2 contains a standard bookwork related to the embedding theorems, see parts  $(a_i)$ ,  $(b_i)$ , although some papers did not present full answer missing certain assumptions on the boundary of domains, etc. The part  $(a_{ii})$  is a variation of proving inequalities by the compactness method.

The main point was to show that the sequence of scaled functions converges strongly in a Sobolev space in order to use the trace theory for taking the limit on the boundary of the domain. Not all students indicated this in the paper. In part  $(b_{ii})$ , almost all students gave a correct answer saying the required embedding is not true in dimension 4. However, to justify this, one needs to find a counter-example, which is the main point of the question.

#### **C4.6 Fixed Point Methods for Nonlinear PDEs**

Question 1: solved by six out of seven candidates. Parts (a) was mostly bookwork and very well solved and also the new part, which is a simple consequence of the definition of the retraction, was correctly answered by nearly all students.

Part (b) was a variation of the proof of Schauder, but required a very careful compactness argument. It was hence not surprising that while the first part of the proof, using a reduction to a finite dimensional setting and Brouwer, was well solved by quite a few students, this second part caused more difficulty and was only successfully tackled by one student, though several others obtained partial points also on that bit of the question.

The first parts of (c) were very well solved, and while the new, and quite difficult part (iii) of the question, was fully solved only by one student, many students got partial marks.

Question 2: Solved by three out of seven candidates. The first part of (a) was bookwork and correctly solved by all, while (ii) was new and was not solved correctly by anyone, as all the candidates attempting this question tried to use directly that  $T - Id$  has fixed point rather than rewriting the equation  $Tx = y$ , for a given  $y$ , into a fixed point problem and only then applying Schauder. Part (b) was bookwork/seen exercise, and part (c) was a typical application of Schauder, with the last bit about uniqueness requiring a careful distinction on the sign of the parameter and an application of the maximum principle.

Question 3: Solved by five out of seven candidates: Part (a) was mostly bookwork and that was well solved, and while most students commented that in the lecture we have only seen that uniqueness holds for strictly monotone operators, few thought to give an example (such as the zero operator) in which uniqueness of solutions fails. Several students did not provide a correct derivation of the variational inequality in (b), in particular wrt. the allowed choice of test functions and parameters. The rest of parts (b),(c) were mostly well solved. Part (d) was new and while it was fully solved only by one student, several students obtained partial points.

#### **C4.8 Complex Analysis: Conformal Maps and Geometry**

Majority of the candidates attempted Questions 1 and 3. Question 1 turned out to be the easiest but many struggled past (c).

In Question 3 all candidates failed to see the connection between (a)(ii), (b)(i) and (c). In Question 3 (c) many tried to use triangle inequality to estimate the power series which is

not a productive dissection.

### C5.1: Solid Mechanics

Q1: Most students tried this question (10./14) and many of them did very well. The question was mostly theoretical and students who learned the material managed to prove most of the statements. Students showed a good understanding on the basis of nonlinear elasticity and were able to manipulate satisfactorily all computations. Only a few students managed to complete the last part that required a better understanding of the material.

Q2: This question was well answered and probably a better text of the students' ability. Most students could do the main basic steps which were similar to homework problems, but only a few students really understood the last steps of the problem and were able to prove the main results. Some student struggled with the basic formulations of the deformation gradient and divergence in polar coordinates.

Q3: This question was probably the hardest and only 6 students tried it. The first two parts of the question should have been easier and straightforward but many students did not know the correct definition of the Young's modulus and did not obtain the correct result. The last part was more involved and a couple of students managed to do most of the problem.

### C5.2: Elasticity and Plasticity

Q1: This question was only attempted by seven candidates. In general it was poorly done. Attempts at the bookwork derivation of the Euler strut equation were particularly poor, with most candidates attempting a global force balance, rather than repeating the derivation of lectures to incorporate the force from gravity. There was also some surprising confusion about the number of boundary conditions that are required for a second-order differential equation. Those candidates who advanced to part (c) generally obtained an appropriate condition on the dimensionless beam length for buckling  $\Lambda$ . In part (d), no candidate was able to use the answer of part (b) to infer that  $\mathcal{P}_c \sim \pi^2/4\Lambda^2$ .

Q2: This question was attempted by every candidate. Part (a) was generally done well, though there were a number of attempts that incorrectly stated the vector identity for  $\vec{a} \wedge (\vec{b} \wedge \vec{c})$ . Part (b) was also generally well done, though some candidates were careless with the directions of travel of the reflected waves and/or the different constant vectors  $\mathbf{a}$  that are required for  $P$ - and  $S$ -waves. Candidates who had time to attempt the final part of the question generally produced a good answer.

Q3: This question was generally popular and was, on the whole, well done. However, candidates should generally be more careful to justify the form of the Tresca condition appropriate to axisymmetry; similarly, a brief justification for why a particular choice of sign for  $\tau_{rr} - \tau_{\theta\theta}$  in the plastic region would be helpful to avoid answers going awry.

### C5.3: Statistical Mechanics

The first question was the most standard, but the derivation of conservation laws from the Boltzmann equation caused some difficulties. The second question on condensation had

been scampered through in lecture, and was algebraically intricate, but generally well done. The third, proving a lower bound for Boltzmann's H function, was also intricate but very well done.

### C5.5: Perturbation Methods

Overall students performed well on the examination.

Question 1. A commonly attempted question, overall answered very well with the integrand singularity in the final part not derailing students, who typically noted it was integrable and proceeded with an approach based on Laplace's method. Marks were more usually lost by not accurately determining the order of the asymptotic corrections.

Question 2. This was rather unpopular and seriously attempted only by a very small minority, who did rather well and were well rewarded in the early stages, though matching the boundary layer at infinity did challenge most of the students.

Question 3. This was extremely popular, though managing the calculation complexity in final part did differentiate the attempts.

### C5.6: Applied Complex Variables

- Q1: Part (a) was mostly done quite well but some candidates had errors with some of the intermediate mappings required and therefore ended up not being able to reproduce the given solution. Most candidates realised the mapping of the hodograph plane in part (b) was the same as that needed in part (a), but some stubbornly continued with their own incorrect mapping and therefore got quite stuck with part (c). Part (b) was well done apart from that. Part (c) caused the most difficulties, but a number of candidates obtained the correct expression.
- Q2: Some candidates did much more work than was required in part (a), using integral expressions for  $w$  itself as well as  $w/\tilde{w}$ ; explanation of where the function  $H$  comes, even brief, was lacking in some cases. Part (b) was generally done well, although some candidates gave incorrect definitions of the square root (giving imaginary values on either side of the branch cut). Part (c) was found the most challenging, with only one or two candidates correctly calculating the contour integral round the large circle (most had it being zero, or made up more exotic expressions to try to arrive at the given solution in the case  $c = 1$ ). Part (d) was done quite well, the connection to the earlier parts being noted.
- Q3: Part (a) was bookwork plus a standard application of the residue theorem; however it seemed to be found tricky by many candidates. Many tried to close the contour to find  $G_+$  in the upper half plane rather than the lower half plane. Part (b) was mostly done very well; a surprising number of candidates failing to note that the required 'splitting' of the right hand side had already been achieved in (a). Part (c) was



not completed correctly by any candidate, although a number came close. A common misconception was to discuss a ‘residue’ at the branch point.

### **C5.7: Topics in Fluid Mechanics**

Q1 was done by all students. All parts except the final rider asking about the significance of the similarity solution was answered by at least one student. No part seemed particularly difficult, though some students had difficulties getting all the aspects of the bookworks in 1(a) complete and consistently written up.

Regarding their second question, students split between Q2 and Q3, with the latter being taken more frequently

Q2 was perhaps the most novel question and therefore was only taken by a few students, and no student got all parts right. In particular part c) was seen as difficult.

Q3 was again done mostly well by those who attempted it, with perhaps the major challenge coming from the bookwork (which focussed on the momentum balance rather than the mass conservation equation as in previous years) and details of the reduction of the system in part c).

### **C5.9: Mechanical Mathematical Biology**

Overall students performed well on the examination.

Question 1. This question was attempted by all students. Failing to control the complexity of the calculation in the final part of the question did see many, but not all, students lose marks.

Question 2. Students found the final parts of this question difficult. The earlier parts of the question provided the constraints required to make progress in the later parts, though this was often not noticed.

Question 3. This was not popular though those who undertook the question generally did very well.

### **C5.11: Mathematical Geoscience**

Q1: This was the most popular question and was attempted by most candidates. It was mostly done well, although some candidates overly complicated the algebra of part (a) and surprisingly many made a hash of the non-dimensionalisation in part (b). Part (c) was well answered on the whole, although many candidates jumped straight to the quasi-steady evolution on the  $O(1)$  timescale without discussing the initial transient evolution of  $p$ . No-one produced a completely satisfactory sketch of the evolution of  $p(t)$ .

Q2: This question was found to be the most challenging. Surprisingly many candidates

struggled with the linear stability in part (a), which was almost identical to an example in lectures and on the problem sheets. The first part of (b) was well done, but most candidates would have benefited from thinking more graphically, and many attempts were much more involved than required. Part (c), which was new and certainly harder, was answered well by two or three candidates, although several others managed to explain the expression for the wavelength.

Q3: This question proved to be relatively straightforward. Part (a) and (b) were answered completely by most who attempted this question. Part (c) was new, and required some broader thinking, but it was mostly well done. Some explanations of the thermal boundary conditions were confused, with a number of candidates suggesting that the presence of the subglacial water layer makes the base insulating.

### C5.12: Mathematical Physiology

Question 1: This was the least popular question. For part (a), many struggled to derive the  $\partial I/\partial z$  term. Some candidates were confused about the dimensions of  $R$  and  $C$ . Part (b) (i) and (ii) were generally well done, though some candidates failed to correctly determine  $l$ . Part (c) was well done.

Question 2: Part (a). Many candidates did not give the correct biological meaning of  $r$ ,  $-kc$ ,  $-k_s c_s$ . In (c)(i) the linear stability calculation was well done by the majority of candidates. Marks were deducted if the direction of the fast portion of the trajectory was incorrect (some candidates indicated horizontal portions of the trajectory).

Question 3: The majority of candidates did not capture the feedback between the number of circulating red blood cells and blood oxygen levels, and the subsequent control of stem cell commitment to the red blood cell lineage. In part (c)(ii) some difficulty was encountered in finding approximate formulae for  $|f'(\xi^*)|$  when  $0 < \delta \ll 1$  and when  $\delta \gg 1$ , mainly due to not correctly finding approximate expressions for  $\xi^*$  in these limits. While many candidates were able to show that  $f'$  was monotonic with respect to  $\xi^*$ , some candidates did not go on to make the connection between  $\xi^*$  and  $\delta$ , and so could not indicate graphically how  $|f'(\xi^*)|$  varies with  $\delta$ .

### C6.1: Numerical Linear Algebra

Q1 on orthogonalities and the SVD was attempted by about 3/4 of the candidates and there were correspondingly a range of scores. There was some confusion over permutations in part (c) and very few made much headway on the final part (e), though some did correctly complete this.

Q2 on simple iteration and SOR was attempted by just less than 2/3 of the candidates and also attracted a range of scores. Many incorrectly assumed the eigenvalues of  $B$  to necessarily be read in part (b) and only a few correctly completed the final part (e).

Q3 on the conjugate gradient method contained the most bookwork but still several candidates put forward incorrect arguments for various parts. Only one or two were able to correctly answer the final part (b)(iv).

## C6.2: Continuous Optimisation

The students have done very well on the exam this year, showing good understanding of both theory and practical examples. In particular, questions 1 and 2 proved popular. The performance on question 2 was consistently high.

## C6.3 Approximation of Functions

Question 1: About half the students attempted Question 1.

(a) Most students who attempted Question 1 got this part correct. The straight-forward solution is to specify two functions  $f$  and  $g$  for which  $B^0(f + g) \neq B^0(f) + B^0(g)$ . Several different examples were provided, and in most cases the best approximations were specified correctly. Some students attempted a more general argument as to why it should be impossible to prove linearity, which was generally less successful.

(b) A majority of students who attempted Question 1 provided more or less correct answers. Some small number of marks were deducted for proofs that neglected to motivate certain steps, or for making incorrect deductions about the permissible range of  $\alpha$ .

(c) This question was skipped by many students. Among the attempted answers, a fair number of marks were deducted for faulty arguments, although there also several perfect answers.

Question 2: About half the students attempted Question 2.

(a) These scripts showed evidence that most students who attempted this question understood the general ideas. Some marks were lost due to incorrect calculations and/or deductions about convergence of series.

(b) Most students described a correct method for evaluating the numbers  $(d^n)$ . Fewer students correctly completed the calculation. Most students also correctly observed that uniform convergence is impossible as the limit function is not continuous. Not many scripts identified that the partial sums converge in a weighted  $L^2$ -norm.

Question 3: Almost all students attempted Question 3.

(a) Most scripts gave correct proofs.

(b) Most scripts gave more or less correct proofs for this statement.

(c) The answers to this question were mixed. Most students described a correct idea for how to use aliasing to construct the interpolants  $L^4(f)$  and  $L^7(f)$ . There were a fair number of errors in evaluating the indices for the different cases, but as long as the general idea was correct, this led to only very minor loss of marks. There were many correct answers for the  $L^2$ -projections as well, but also a fair number of faulty arguments. (Some students confused  $L^n(f)$  with the  $L^\infty$  best approximation  $B^n(f)$  from Question 1.) Computing the distances between the original function and the approximant proved surprisingly difficult, with many incorrect answers and unnecessarily complicated computations.

(d) Most answers to this question were in principle correct. There were many flawless ones, and among the ones that lost marks, this was typically only a very minor loss due to getting some scaling factor or sign wrong.

## C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question contained some new material that was not covered in previous years. As a result, many students avoided this question for the others. However, it did reveal a good spread of abilities across the students who did attempt it. In Q1 (a) (i), several students struggled with applying integration by parts twice to derive the weak form of the biharmonic equation. Very few were able to deduce the boundary conditions from the variational form in (b) (i) and as a result struggled with the rest of part (b). The material in part (c) was emphasised heavily in the lecture notes and lectures, but several students defined the Lagrange finite element instead of the Argyris element, and could not explain that Argyris was advantageous as it is  $H^2(\Omega)$ -conforming. Almost no students attempted (d), even though it is a relatively straightforward extension of material covered extensively in the lectures to three dimensions.

Q2: This question was the most similar to those of previous years, but with the small twist that the coercivity constant depended on a parameter in the equation. The pattern of marks for this question was quite bimodal, with those students attaining fewer marks often getting basic bookwork wrong such as the statement of Galerkin orthogonality in (b)(i) or the standard interpolation error bound for Lagrange finite elements in (b)(iii). Only a small number could correctly structure the duality argument requested in (d).

Q3: This question introduced novel material on noncoercive problems. This was compensated by the similarity of parts (a) and (b) to the last problem sheet of the course. Nevertheless, some students struggled with calculating Fréchet derivatives, or correctly stating Newton's method, or proving that satisfying Lax–Milgram implies that the conditions of Babuška's theorem hold. Many students were unable to identify that the Euler–Lagrange equation for (T) would be noncoercive for large  $k$ . Several students stated the Newton–Kantorovich theorem in (b) (vi) instead of answering the question. Part (c) was very successful in identifying the best candidates, as it required some novel thought. Strangely, many of the candidates did not use the hint provided, which instructed them on how to begin the argument.

## C7.4: Introduction to Quantum Information

### Question 1

Parts (a) - (c) were bookwork with very few marks lost. Most students struggled with part (d) and calculating the action of the Grover iteration operator on the states. Good attempts at parts (e) and (f).

### Question 2

Fairly well done question. Parts (a) and (b) were bookwork and posed no difficulty. Neither did part (c), though only a few candidates used stabiliser generators to prove it. In part (d) almost all students erroneously thought the state was entangled. Good attempts at (f) and (g). However, only a handful of students considered calculating the reduced density matrix in part (g).

### Question 3

This was the most popular question on the paper, probably owing to the familiar bookwork in parts (a) and (b), but it also had the lowest average mark. Many students struggled with part (c). There were various attempts at part (d). Some candidates had the right idea but

failed to spot the block structure of the matrix and got lost in the algebra of diagonalisation.

### **C7.5: General Relativity I**

Hardly any of the students tried question 3 and those that did were unable to make progress on the final part. The answers for question 2 were generally very good, a few students even succeeded in deriving the Reissner-Nordstrom solution. Question 3 attracted many students as well, but few managed to solve the final part.

### **C7.6: General Relativity II**

Q1: This was not very popular. Six candidates attempted it and did very well (with an average mark of 19/25 raw marks).

Q2: This question attracted 18 attempts and the average mark was 17.5, which is very good. Most points were lost in part c of the question and in part b where students did not produce an accurate space time diagram.

Q3: This question attracted 16 attempts and the average mark was 16.5, which is good. Students were not able to obtain the answer for part c(i) correctly.

### **C8.1: Stochastic Differential Equations**

Question 1 was very popular and all candidates attempted it. Everybody managed to show 1a and nearly all candidates managed to show 1b (some gave convoluted answers and tried to reprove Novikov for this particular example). 1c led to long calculations that led often nowhere, especially for 1C(ii) few realised that the integral for  $A_t$  can be split up and the explicit solution for the SDE can be used. Question 2 and Question 3 were approximately equally popular. A common mistake was failing to specify in 2a(i) that one needs the left-derivative, and in 2b) to ignore that one needs a countable union of null-sets. Nobody made substantial progress on 2c(ii) and few made progress on 2c(i). Similarly, most candidates managed to show 3a and 3b, but few candidates made any progress on 3c) beyond 3c(ii).

### **C8.2: Stochastic Analysis and PDEs**

Question 1:

This question was the least popular and proved more challenging for those that attempted it. The first parts were bookwork. The second section on the resolvent was more challenging than anticipated with many not even able to show (b)(i). The final part only had one good attempt.

Question 2:

This was the most popular question and saw a wide range of marks. The bookwork was well done. The proof of the convergence to Brownian motion in (b) was mixed, with many not getting the correct value for the time change or not establishing all the conditions of the convergence theorem. There were a number of good attempts at the final part.

Question 3:

The final question was quite popular and generally well done. Part (a) was bookwork and well done. For part (b) the standard martingale proof of the representation was not easy to implement. The final part was solved completely by 2 people.

### C8.3: Combinatorics

**Question 1:** Parts (a) and (b), being fully bookwork, were done well. In part (c) many candidates tried to once again apply the LYM Inequality which could not lead to a complete solution. Even though most candidates could find the argument for  $k = 1$ , not many managed to spot how it could lead to an analogous solution in the general case, where a single set in the  $k$ th layer “blocks”  $\binom{n-k}{n-2k} = \binom{n-k}{k} \sim \binom{n}{k}$  sets in the  $(n - k)$ th layer. In part (d), only a few candidates spotted that the “appropriate choice” suggested by the hint could be the larger (or equivalently, the smaller) set from every pair  $(A, B_A)$ .

**Question 2:** This question was a popular choice among the candidates. Parts (a) and (b) were done well, and the choices of the proof of the Erdős-Ko-Rado Theorem varied between candidates, with both the circular permutations method and the approach using the Kruskal-Katona Theorem being present in large numbers (although the circular permutations proofs often lacked some details). Part (c) was done very well, but part (d) has caused more trouble. Some candidates successfully adapted the Kruskal-Katona proof of Erdős-Ko-Rado to this problem, although that was not necessary for “ $n$  large enough”; indeed, knowing that there must be some disjoint  $A_1, A_2 \in \mathcal{A}$  we can easily bound the number of possible  $B \in \mathcal{B}$  that intersect both of them by  $r^2 n^{r-2} \ll \binom{n}{r-1}$ .

**Question 3:** Part (a) was mostly done well, although some candidates started the summation from  $i = 1$  in the bound on the size of the family in question. In part (b) most candidates successfully showed that the characteristic vectors of the sets  $A_i$  are linearly independent by taking a product with the characteristic vectors of consecutive sets  $B_i$ . Part (c) caused no trouble, while part (d) was done well by a few students, who successfully generalised the case  $m = 1$  presented in the lectures. However, some candidates only showed that at least  $nm$  hyperplanes are necessary, without showing that this number is also sufficient.

### C8.4 Probabilistic Combinatorics

**Question 1:** This question was not a popular choice among the candidates. Surprisingly many marks were lost in parts (a) and (b), despite them being fully bookwork. Part (c) was a straightforward application of Chebyshev’s inequality and part (b) but still caused some confusion. Part (d) was generally done well, although some candidates did not observe that we need to look at the expectation of the sum of the numbers of copies of red and blue cliques to be able to use the vertex-deletion argument. Finally, in part (e) both subparts (i) and (ii) have generally been done well, but subparts (iii) and (iv) proved to be rather challenging.

**Question 2:** This question was the most popular choice. Both parts (a) and (b) were done well, and many candidates offered interesting examples to prove that the condition  $p(d + 1) \leq 1$  is not in general sufficient for the SLL to hold. Part (c) was again done well, with only a few candidates struggling to find a correct bound on the outdegrees in the dependency graph. The main difficulty in part (d) was finding the correct random choice, but most candidates realised that selecting a random point from every colour class led to a straightforward application of the SLL. Other difficulties encountered by the candidates in this part included understanding that the colouring is fixed (rather than random), and then once again bounding the outdegrees in the dependency graph.

**Question 3:** This question was once again a popular one. Parts (a) and (b) have mostly been done very well. In part (c) most candidates had a good general idea of the argument, but some details were often missing in their proofs. This was sometimes caused by trying to not only look at the random variables  $X_{w_1, w_2}$  as suggested in the hint, but also considering paths of length 1 and 2 between  $u$  and  $v$ , which was unnecessary. In the application of the Janson's inequality, many candidates found bounding the value of  $\Delta$  challenging. For example, there are  $(n - 2)(n - 3)$  choices of  $w_1, w_2$  rather than  $\binom{n-2}{2}$ , as the order of vertices matters here. Also,  $(w_1, w_2) \sim (w_3, w_4)$  if  $w_1 = w_3, w_2 \neq w_4$ , or  $w_1 \neq w_3, w_2 = w_4$ , or  $w_1 = w_4, w_2 = w_3$ , resulting in  $(2(n - 4) + 1) = 2n - 7$  pairs  $(w_3, w_4)$  contributing  $p^5$  each to  $\Delta$  for every pair  $(w_1, w_2)$ . Finally, some candidates did not observe that an application of the union bound over all pairs  $u, v$  is necessary for the argument about the diameter of the graph to hold, and that this is where  $c > 2$  becomes important.

### Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC6 - Graphical Models SC7 - Bayes Methods

### Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science Categories, Proofs and Processes Computer Animation

## F. Comments on performance of identifiable individuals

Removed from public version.

## G. Names of members of the Board of Examiners

- **Examiners:**

Dr. J Woolf (External)  
Prof. G Chen (Chair)  
Prof. A Goriely  
Prof. M Kim  
Prof. C Howls (External)  
Prof. J Ball  
Prof. A Ritter

- **Assessors**

Prof. Samson Abramsky  
Prof. Konstantin Ardakov  
Prof. Ruth Baker  
Prof. Charles Batty  
Prof. Dmitry Belyaev  
Dr Andreas Braun  
Dr Coralia Cartis  
Prof. Dan Ciubotaru  
Prof. Samuel Cohen  
Prof. Andrew Dancer  
Prof. Chris Douglas  
Prof. Xenia de la Ossa  
Prof. Cornelia Drutu  
Prof. Artur Ekert  
Dr Patrick Farrell  
Prof. Victor Flynn  
Prof. Andrew Fowler  
Prof. Eamonn Gaffney  
Prof. Ben Green  
Prof. Peter Grindrod  
Dr Stephen Haben  
Prof. Ben Hambly  
Dr Heather Harrington  
Dr Andre Henriques  
Prof. Ian Hewitt  
Dr Chris Hollings  
Prof. Samuel Howison  
Prof. Ehud Hrushovski  
Dr Daniel Isaacson  
Prof. Dominic Joyce  
Prof. Peter Keevash



Prof. Kobi Kremnitzer  
Prof. Minhyong Kim  
Dr Robin Knight  
Prof. Marc Lackenby  
Prof. Alan Lauder  
Prof. Per-Gunnar Martinsson  
Prof. Lionel Mason  
Prof. Kevin McGerty  
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